

**Final Exam**

**All of your answers/solutions/computations must be written on the provided Answer Sheet.**

**Matching**

*Match the following vocabulary terms with the best description.*

1. **A.** a cipher system that replaces the letters in a message with other characters
2. **B.** a prime number that can be expressed as one less than a power of two
3. **C.** proved that positive integers can be expressed as the sum of at most 3 triangular numbers
4. **D.** a graph where every vertex is adjacent to every other vertex
5. **E.** an edge that begins and ends at the same vertex
6. **F.** a cipher system that scrambles the ordering of the letters in a message
7. **G.** conjectured that positive integers can be expressed as the sum of two primes
8. **H.** a pair of positive integers whose greatest common factor is 1
9. **J.** an edge that when removed will cause the graph to become disconnected
10. **K.** a graph where a path exists from every vertex to every other vertex
11. **L.** a positive integer that is only divisible by itself and the number 1
12. **M.** a graph where the sum of all of the degrees of the vertices is even
13. **O.** proved that there are an infinite number of primes

**Multiple Choice**

*Read each question and then select the most correct response or responses.*

8. In the graph shown at right, which of the following vertices is a cut vertex?  
**A.**                      **B.**                      **C.**
9. The graph shown at right is planar.  
**A.** True                      **B.** False                      **C.** depends
10. The graph shown at right is not complete. How many more edges need to be *drawn* to make the graph complete?.  
**A.**                      **B.**                      **C.**
11. The graph shown at right does not contain an Euler path. Which of the following edges, when added to the graph, will allow an Euler path?  
**A.**                      **B.**                      **C.**                      **D.**
12. The number 241 is congruent to the number 89 in a modular 19 arithmetic system.  
**A.** True                      **B.** False                      **C.** Depends                      **D.** Impossible to tell
13. Which of the following is not a triangular number?  
**A.**                      **B.**                      **C.**                      **D.**
14. \_\_\_ proved that  $n^p \equiv 1 \pmod{p}$  whenever  $p$  is prime.  
**A.**                      **B.**                      **C.**                      **D.**
15. Which of the following are characteristics of an effective cipher system:  
**A.**  
**B.**  
**C.**  
**D.**  
**E.**

### Free Response – Procedural/Computational

Answer the following being sure to show the work you did to arrive at your answer whenever possible.

16. Using only addition & multiplication, solve  $42 + 8x \equiv 10 \pmod{15}$
17. The Highwired Cipher System is an affine cipher in mod 26 that is combined with backwards writing. Encode the word HELP using additive key 8 and multiplicative key 3. Be sure to clearly show each of the steps you take in the enciphering process.
18. Provide mathematical proof/evidence that the number 2,011 is prime.
22. Kevin decided to simplify his fashion life by devising a systematic way for choosing his daily attire. Starting January 1<sup>st</sup> of each year, he will wear the items listed first in the chart at right. On each subsequent day, he will wear the next item at the list. When he gets to the end of a particular list, he will move back to the top of that list. Use modular arithmetic to predict Kevin's attire on his birthday which is on June 3<sup>rd</sup>

Pants/Shorts	Shirt	Shoes
Cargo Pants	Red Polo Shirt	Doc Martins
Acid-Washed Jeans	Brown T-Shirt	Sandals
Khaki Pants	Green Dress Shirt	Skechers
	Blue Thermal Shirt	Sneakers
	Gray Bellarmine T-Shirt	

Jan	Feb	Mar	Apr	May	Jun
31	28	31	30	31	30

### Conceptual/Analysis

Answer the following using complete grammatically correct sentences.

20. This question will focus on prime numbers (definition, frequency, importance, algorithms to test, etc.)
21. Read the following passage:

What is mathematics? If you ask this question of the first person you meet on the street, you will most likely hear that "mathematics is the study of number." If you insist that your respondent be more specific, you may elicit the suggestion that mathematics is "the science of numbers." . . . How does today's mathematician answer the question, "What is mathematics?" The most common answer is that mathematics is the science of patterns.

—An excerpt from *The Math Gene* by Keith Devlin

After reflecting on this passage and your experience this semester in this class, write a coherent paragraph addressing the following questions:

- \* How does your view of mathematics compare with the idea expressed in the above passage?
- \* Has your view of mathematics changed over the course of this semester? Explain.